

Controlling the toroidal excitations in metamaterials for high- Q response

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The excitation of toroidal multipoles in metamaterials was investigated for high- Q response at a subwavelength scale. In this study, we explored the optimization of toroidal excitations in a planar metamaterial comprised of asymmetric split ring resonators (ASRRs). It was found that the scattering power of toroidal dipole can be remarkably strengthened by adjusting the characteristic parameter of ASRRs: asymmetric factor. Interestingly, the improvement in toroidal excitation accompanies increasing of the Q -factor of the toroidal metamaterial, it is shown that both the scattering power of toroidal dipole and the Q -factor were increased near one order by changing the asymmetric factor of ASRRs. The optimization in excitation of toroidal multipoles provide opportunity to further increase the Q -factor of toroidal metamaterial and boost light-matter interactions at the subwavelength scale for potential applications in low-power nonlinear processing and sensitive photonic applications.

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I. INTRODUCTION

Optically resonant modes are of fundamental importance in realizing enhanced light-matter interactions for their binding ability of light fields in both the spatial and time domain. For the spatial domain binding, it is highly desirable to localize light in a possibly small volume for strong fields. Conventional dielectric microcavities have been reduced from the “classical” regime to “quantum” regime where wave effects become dominant.¹ And plasmonic excitations in metallic structures have been exploited for realizing light localization on deep subwavelength scale to join together the well-developed nanoelectronic technology.² While for the time domain binding, a dimensionless physical quantity, quality factor (Q -factor), is always adopted for measuring energy damping relative to the stored energy of the oscillating, i.e., the photon lifetime of a resonant mode.

Metamaterials and metasurfaces,^{3–6} which allow light continually interact with medium in the resonant mode during its slow damping, are promising for the enhancement of light-matter interactions at a subwavelength scale. Various plasmonic metamaterials have been proposed for achieving high Q -factor response, for example, the trapped-mode, which is weakly coupled to free space, was suggested to be excited by introducing symmetry breaking in the shape of structural elements for realizing sharp spectral response.⁷ The trapped mode was later demonstrated in similar metamaterials and the resulted high- Q resonance^{8,9} was utilized for applications in ultrasensitive spectroscopy and sensing,¹⁰ high- Q filtering,^{11,12} optical switching,¹³ nonlinear optics,¹⁴ and lasing spasers.¹⁵ Recently, the excitation of toroidal moment¹⁶ in felicitously designed metamaterial was suggested as a new solution for achieving high- Q resonance.¹⁷ The toroidal excitations are different from electrical and magnetic excitations in traditional multipole expansions,^{18,19} it was found that higher Q response (in comparison with the original magnetic dipolar mode) in a metamaterial can be achieved by taking advantages from the weak free-space coupling of the toroidal dipolar excitation.^{20–22} Later, the toroidal excitations were studied in oligomer nanocavities,²³ coupled nanodisks,^{24–26} coupled dielectric/metallic rods.^{27,28} The exotic toroidal response were also investigated for optical manipulations in polarization control,^{29–32} high- Q subwavelength cavities,^{33–37} invisibility,^{38–40} and optical force.⁴¹

In this paper we show how to achieve higher Q -factor in a planar metamaterial by controlling the toroidal excitations. The unit cell, also called as meta-molecule, of the metamaterial

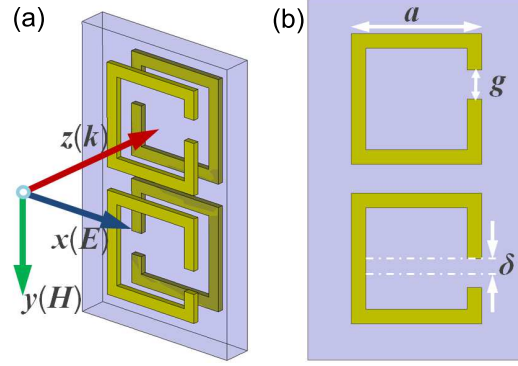


FIG. 1. (a) Schematic of a meta-molecule of the toroidal metamaterial, the metamaterial is illustrated by normal incident wave with the electric polarization is along x -axis. (b) Top view of the meta-molecule with the geometric size being labelled by white arrows and black letters.

is comprised of four asymmetric split ring resonators (ASRRs). The magnetic resonance of the ASRRs can be excited by electromagnetic wave with polarization direction perpendicular to the gap of the ASRRs. The near-field coupling^{20,42–45} of these magnetic resonators will induce a toroidal moment dominated fundamental resonance in the planar metamaterial. In the study, we find that the Q -factor of the planar metamaterial can be improved by tuning the asymmetric factor of the ASRRs, both the magnetic mode of the ASRR and near-field coupling between ASRRs in the meta-molecule are sensitive to the asymmetric factor for that the magnetic mode originate from the symmetry breaking to external stimulus.⁷ The fundamental resonance is of Fano-lineshape and the modulation depth of the resonance is nearly unity, which makes the toroidal metamaterials suitable for possible applications in ultra-sensitive sensors. Toroidal geometry together with the Fano resonance made high- Q response will have enormous potential applications in low-threshold lasing, cavity quantum electrodynamics and nonlinear processing.

II. RESULTS AND DISCUSSIONS

The meta-molecule of the designed toroidal metamaterial is presented in Fig. 1, as can be seen from the figure, the metamaterial is comprised of two patterned metallic layers and a dielectric spacer layer, the metallic layers are patterned into arrays of asymmetric split ring resonators (ASRRs). Four ASRRs are properly arranged for controlling the structural

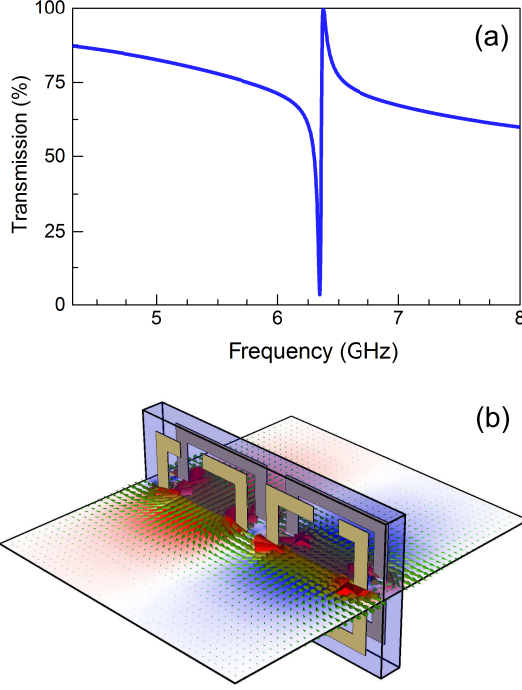


FIG. 2. (a) Calculated spectrum of a designed planar metamaterial with toroidal moment dominated Fano resonance. The asymmetric parameter was set to be: -4 mm. (b) Calculated spatial distributions of vectorial magnetic field over the color map of z -component magnetic field on the central plane of a meta-molecule at the peak frequency of the transmission spectrum.

symmetry of the meta-molecule: pairs of ASRRs on same layers are of mirror symmetry about the xz -plane; pairs of ASRRs on different layers are of 2-fold rotational symmetry about the y -axis. The lattice constants along x - and y - directions are 5 mm and 10 mm, the permittivity of the 0.8 mm-thick dielectric spacer is 2.55. The width of metal strip is 0.5 mm, the outer width of the ASRRs is $a = 4$ mm, the distance between the ASRRs is 0.8 mm. The asymmetry of the ASRRs was introduced in the square rings by opening gaps of size $g = 1$ mm positioned δ from the center of the rings, the undetermined asymmetric factor δ is crucial in the control of the toroidal excitations as we will see. We performed all numerical calculations with a finite-difference-in-time-domain (FDTD)⁴⁶ electromagnetic solver.⁴⁷ And we note that throughout our study the metamaterials are illuminated by x -polarized (the electric field \mathbf{E} is along x -axis) electromagnetic waves as illustrated in Fig. 1(a).

The ASRRs in the toroidal molecule are in close proximity, it is well known that the mutual interactions or couplings⁴⁸ between these building blocks play determinative role in

their collaborative response to incident fields. The couplings between the ASRRs in both the in-plane pair and the stacked pair were studied in our previous work.³³ The in-plane ASRR pairs were designed of mirror symmetry with respect to the xz -plane. The resonant frequency of the paired ASRRs is the same as that of ASRRs. Different from the metamaterials with period ASRRs, the mirror symmetry ensures the paired ASRRs strongly coupled to the incident waves, which makes the trapped magnetic mode of ASRRs can be well excited. To mimic the head-to-tail shaped magnetic morphology of toroidal dipolar excitation in the bulk metamaterial, two layers of metallic ASRRs were stacked to couple the magnetic mode in ASRRs, the paired ASRRs on top and lower layers in the meta-molecule was designed to be of 2-fold rotational symmetry with respect to the y -axis. This vertical coupling configuration provides a remarkable overlap between the magnetic dipoles of ASRRs on top and lower layers, indicating strong magnetic coupling. The coupled magnetic modes in ASRRs show in-phase and out-of phase states at low and high frequencies, respectively. The high-frequency resonance is determined by the out-of-phase vertical coupling (not shown here). Since we mainly focused on the toroidal excitation in this paper, we show in Fig. 2 the transmission around the low-frequency resonance and the on-resonance local field distribution, the low-frequency resonance is determined by the in-phase vertical coupling [see color map of the z -component magnetic field in Fig. 2(b)] together with horizontal coupling under normal electromagnetic incidence made a circumfluent magnetic field or magnetic vortex, which is exactly the picture of toroidal dipolar^{19,20} which shows a subwavelength electromagnetic localization within dielectric substrate in a toroidal configuration.

To quantitatively evaluate the contributions on electromagnetic scattering of the multipoles, we adopted a current density formalization²⁰ by integrating spatial distributed fields in a meta-molecule. In the current density formalization, the radiating power of induced multipoles can be calculated with the formula

$$I = \frac{2\omega^4}{3c^3} |\mathbf{P}|^2 + \frac{2\omega^4}{3c^3} |\mathbf{M}|^2 + \frac{2\omega^6}{3c^5} |\mathbf{T}|^2 + \dots \quad (1)$$

with electric dipole moment being

$$\mathbf{P} = \frac{1}{i\omega} \int \mathbf{j} d^3r, \quad (2)$$

magnetic dipole moment being

$$\mathbf{M} = \frac{1}{2c} \int (\mathbf{r} \times \mathbf{j}) d^3r, \quad (3)$$

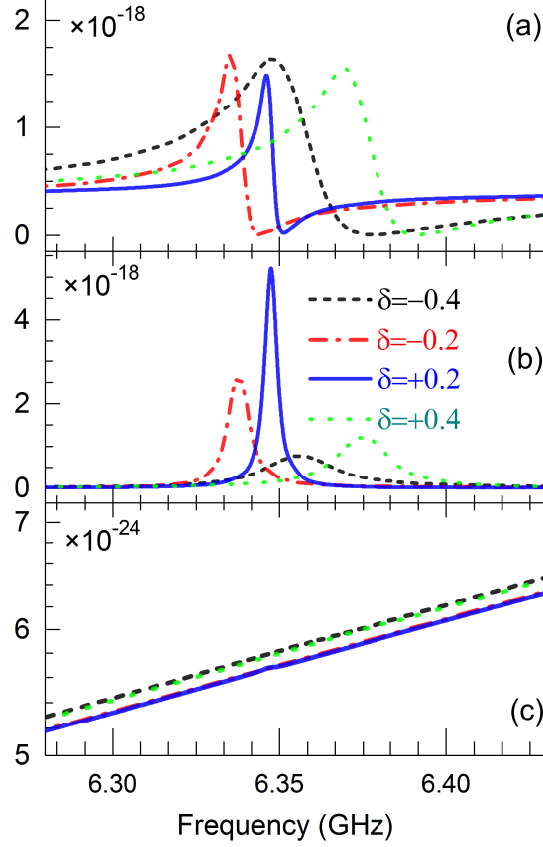


FIG. 3. The radiating power for induced multipoles of the planar toroidal metamaterial with different asymmetric parameter δ : electric dipole (a), toroidal dipole (b), and magnetic dipole (c).

and toroidal dipole moment being

$$\mathbf{T} = \frac{1}{10c} \int [(\mathbf{r} \cdot \mathbf{j}) \mathbf{r} - 2r^2 \mathbf{j}] d^3r, \quad (4)$$

in which \mathbf{j} is the current density.

The radiating power of three mainly contributed multipoles of the metamaterials are shown in Fig. 3: electric dipole (\mathbf{P}_x), toroidal dipole (\mathbf{T}_x), and magnetic dipole (\mathbf{M}_y). We first show the scattering powers of the multipoles for the metamaterial with asymmetric parameter $\delta = -0.4$ mm [dashed black lines, the minus sign ($-$) means the gap is away from the center of the meta-molecule], the electric dipole \mathbf{P}_x shows strong scattering in the whole frequency band, which reveal that the metamaterial is excited by electric component of the incoming wave. The scattering of magnetic dipole \mathbf{M}_y is about five orders lower than that of electric dipole for the structure is hard to couple with parallel magnetic component of the incoming wave. The toroidal dipole \mathbf{T}_x shows considerable scattering around the resonance,

the x -component of the toroidal dipole moment plays dominative role at the transmission peak frequency 6.38 GHz in comparison with electric dipole and magnetic dipole.

The meta-molecule of the toroidal metamaterial is constructed with ASRRs, and the unusual property of the ASRR is the high- Q trapped mode originating from structural symmetry broken. The asymmetric parameter is crucial in the trap mode of ASRRs which is the fundamental element in the formation of toroidal mode. We thus investigated the influence of asymmetric parameters on the toroidal excitation in metamaterial. Figure 3 shows that the scattering power for induced electric and magnetic dipole is only slightly changed when the asymmetric parameter changes. But differently, the scattering power of toroidal dipole changes significantly along with the change of asymmetric parameter: the toroidal moment is stronger with small asymmetric parameters and when the gap is close to the center of toroidal meta-molecule. The trap mode of the ASRR is sensitive to the asymmetric parameter, an ASRR with small asymmetric factor is weakly coupled to external stimuli and shows stronger binding in time domain, then the excited in-phase magnetic mode, or toroidal mode becomes even more strong compared with bigger asymmetric factor.

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Intuitively, the improvement in exciting the toroidal dipole will be beneficial to achieve higher- Q response in the metamaterials for that both the higher- Q with smaller asymmetry and the enhanced toroidal excitations are promising in binding electromagnetic waves. The Q -factor of a symmetric Lorentzian resonance can be calculated from the ratio of cen-

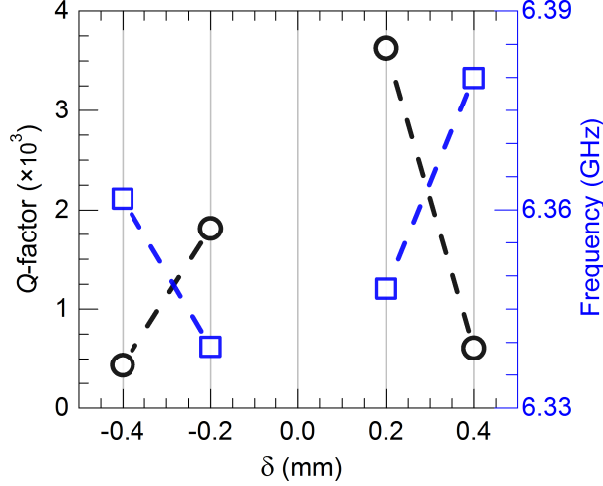


FIG. 4. The resonant frequencies (blue squares) and Q -factors (black circles) of the toroidal metamaterials with asymmetric parameter δ changing from -4 mm to $+4$ mm.

tral frequency and FWHM, which is quite convenient. In the meanwhile, the Q -factor of asymmetric Fano resonance^{49,50} were always got by fitting the spectrum with an Fano formula. Here, we propose to extract the Q -factor from characteristic frequencies (dip and peak frequencies: f_d and f_p) of the Fano resonance, which can be understood as: The Fano-shaped asymmetric resonance originates from interference between a sharp resonance (discrete state) and the continuum (background). The resonant region lies between the dip and peak frequencies, and f_d and f_p are corresponding to the destructive and constructive interferences, respectively. The central frequency of Fano resonance would be $(f_d + f_p)/2$, and the FWHM would be $|f_d - f_p|/2$, then the Q -factor of an Fano resonance can be got in compare with the Lorentzian resonance:

$$Q = \frac{f_d + f_p}{|f_d - f_p|} \quad (5)$$

Figure 4 plots the resonant frequencies (blue squares) and calculated Q -factors (black circles) of the toroidal metamaterials with different asymmetric parameter δ (changing from -4 mm to $+4$ mm) (the calculated Q -factors using Eq. 5 agree well with fitted results with the Fano formula, not shown here). We can clearly see that Q -factor of the toroidal metamaterial increases significantly with small asymmetric parameters and when the gap is close to the center of toroidal meta-molecule, the Q -factor of a metamaterial was increased to 3600 ($\delta = 0.2$ mm) from 432 ($\delta = -0.4$ mm). The variation tendency of Q -factor is

identical to the scattering power of toroidal excitations, which confirms the prediction that the Q -factor of toroidal metamaterial can be raised by optimize the excitation of toroidal dipole.

III. CONCLUSION

In summary, we studied the toroidal excitation in a planar metamaterial comprised of ASRRs, it is shown that the toroidal excitation in a planar metamaterial can be controlled by tuning the asymmetric of ASRRs, and the enhancement in toroidal excitation can remarkably improve the Q -factor of the toroidal metamaterial. The high- Q feature of the toroidal metamaterial and the further improvement on high- Q through geometric optimization may be beneficial to enhance light-matter interactions at a subwavelength scale for low-power nonlinear processing and sensitive photonic application.

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